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DESIGN OF MONOLITHIC CONCRETE FRAME – PRESTRESSED

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DESIGN

OF

MONOLITHIC CONCRETE FRAME - PRESTRESSED

K. P. Milbradt*
T. J. Kofodimos*

I - INTRODUCTION

A project entitled "Prestressed Concrete Frames" was commenced in 1950 for Port Hueneme Construction Battalion Center in California by the Department of Civil Engineering of the Illinois Institute of Technology. The object of the project was to investigate the design and construction problems connected with continuous prestressed concrete frames. The first aim in this respect was to design a prestressed concrete frame for moments similar to those which occur in a continuous concrete structure. The designed frame was constructed and tested in the laboratory to corroborate certain design assumptions and the distribution of prestressing and live load moments. This paper presents a solution to the problems encountered in designing continuous prestressed

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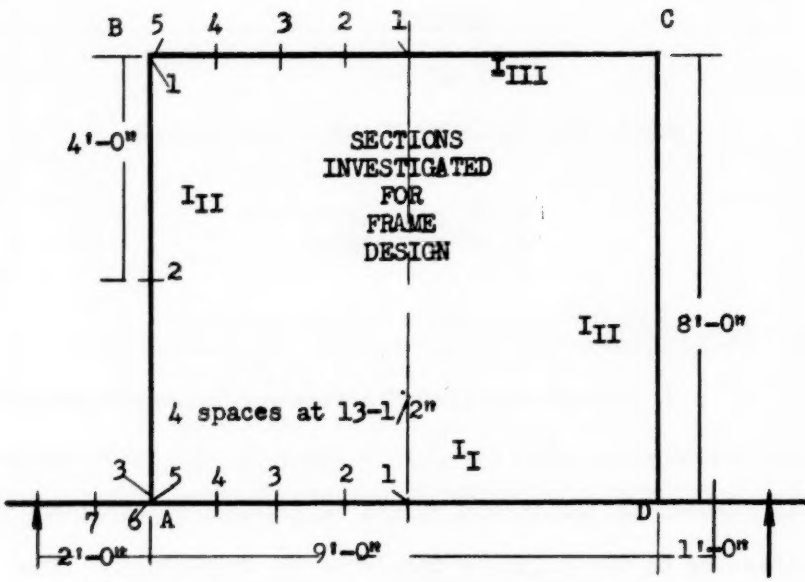


FIG. 1 - Schematic diagram of frame illustrating center-line dimensions and sections used in the analysis.

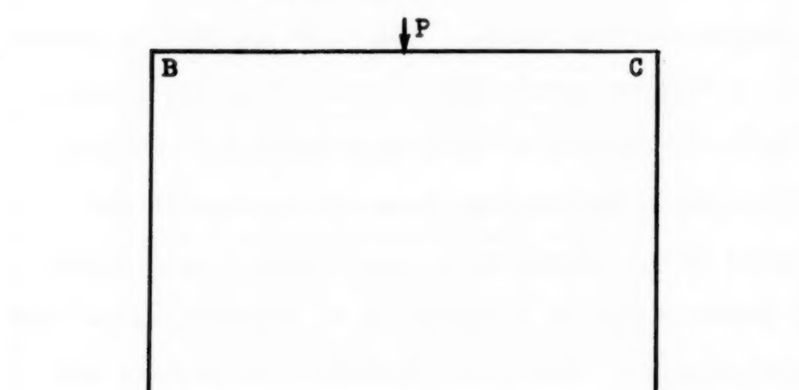


FIG. 2 - Load and support condition creating live load moments M_{L1} and dead load moments M_{D1} in the frame members.

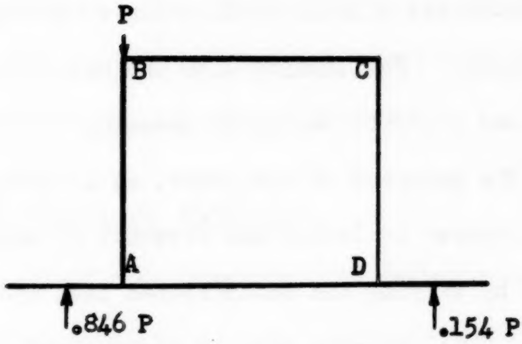


FIG. 3 - Load and support condition creating live load moments M_{L2} and dead load moments M_{D1} in the frame members.

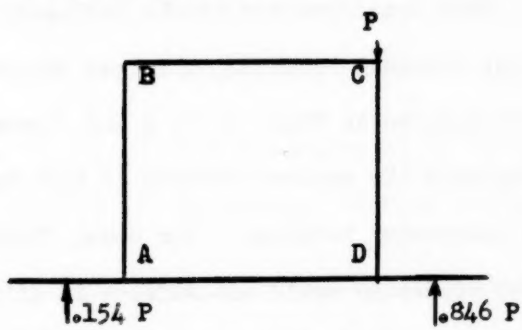


FIG. 4 - Load and support condition creating live load moment M^{1}_{L2} and dead load moments M_{D1} in the frame members.

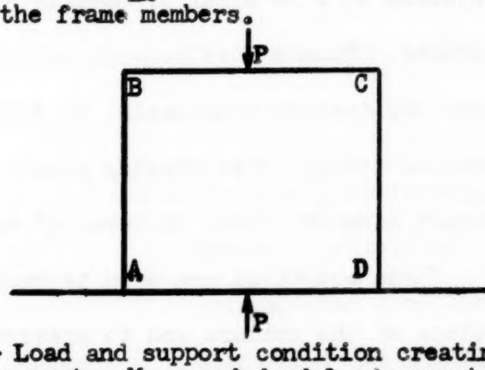


FIG. 5 - Load and support condition creating live load moments M_{L3} and dead load moments M_{D2} in the frame members.

concrete structures with an application to the aforementioned frame. Familiarity with the basic concepts of prestressed concrete design is assumed.

The geometry of the frame, as illustrated in Fig. 1, was chosen to facilitate reversal of moments in the members by varying the concentrated load and support positions. Thus, moments such as encountered in the usual building or continuous bridge were reproduced in the frame. From consideration of the influence lines for moments at critical sections, the load and support positions illustrated in Figs. 2, 3, 4 and 5 were chosen as imposing the desired moments on the frame members for laboratory testing. The usual "trial and error" method of design would not suffice in efficiently obtaining a solution for a continuous structure whose members are subjected to a reversal of moments - for example, this frame. Thus a more general approach involving equations was developed to assist in determining the required section moduli, prestressing forces and prestressing thrust eccentricities in terms of the applied moments. These equations are used to design "critical" sections of the members and to prescribe bounds of the prestressing thrust eccentricity through-

out the members. The derived equations are presented in Appendix "A" and may be used for determinate or indeterminate prestressed concrete structures.

II - DEFINITIONS AND NOTATIONS

Insofar as possible, the definitions and notations for prestressed concrete proposed by the Joint A.C.I. - A.S.C.E. Committee 323 are used.

I,II,III... . . . Denote member I, II, III... respectively; used as a superscript.

1,2,3... j... . . . Denote section 1, 2, 3..., j... of a member; used as a superscript.

F_i . . . Initial prestressing force.

F_o . . . Prestressing force after release.

F . . . Effective prestressing force after deduction of all losses.

F^I . . . Effective prestressing force in member I, after deduction of all losses, when considered constant throughout the member.

F^{Ij} . . . Effective prestressing force after deduction of all losses at section j of member I.

η . . . Ratio F/F_o may be considered constant and equal to 0.85 for post-tensioned members with steel strains of 4.2×10^{-3}. For pretensioned members with the same steel strain, a constant value of 0.80 may be used.

A_c . . . Area of entire concrete (steel area not deducted).

$\left.\begin{matrix} A_s^{I} \\ A_s^{II} \end{matrix}\right\}$ . . . Total steel area, in simply reinforced section of member I, II... .

$\left.\begin{matrix} A_s^{Ijb} \\ A_s^{Ijt} \end{matrix}\right\}$ . . . Area of bottom (top) reinforcement in a doubly reinforced section j of member I.

c.g.c. . . . Center of gravity of entire concrete section.

c.g.s. . . . Center of gravity of steel area.

h . . . Total depth of section.

d ... Effective depth of section.

b ... Width of rectangular section.

I ... Moment of inertia of entire concrete section about c.g.c.

Z_b, Z_t ... Section modulus of bottom (top) fiber referred to the c.g.c.

Z_r ... Ratio Z_b/Z_t.

e ... Cable ordinate as measured from the c.g.s. to c.g.c.

M_D ... Bending moment due to the dead load active when the prestress is being established.

M_L ... Live load moment due to all live loads as well as all additional (superimposed) dead load per unit length applied after the prestress has been established.

k_L^{Ij} . . . Ratio $\dfrac{\min M_L^{Ij}}{\max M_L^{Ij}}$;

k_L is substituted as a positive quantity in the equations of Appendix "A" when min M_L^{Ij} and max M_L^{Ij} are of opposite sign.

k_D^{Ij} . . . Ratio of the dead load moment corresponding to the supporting condition for which min M_L^{Ij} is obtained, to the dead load moment corresponding to the supporting condition for which max M_L^{Ij} is obtained; k_D is substituted as a positive quantity in the equations of Appendix "A" when the dead load moments of this ratio are of opposite sign. In the usual case of fixed supports and constant dead load moment for each section of any member, $k_D = -1$.

α . . . Ratio $\dfrac{\max M_L^{Ij}}{M_D^{Ij}}$ where M_D^{Ij} is the dead load moment corresponding to the supporting condition for which max M_L^{Ij} is obtained.

e_{Do}, e_D . . . Equivalent thrust eccentricity due to the dead load after release and after deduction of all losses in the prestressing force, respectively;

$$e_{Do} = \frac{M_D}{F_o} \qquad e_D = \frac{M_D}{F}$$

e_{Lo}, e_L . . . Equivalent thrust eccentricity due to the live load after release and after deduction of all losses in the prestressing force, respectively;

$$e_{Lo} = \frac{M_L}{F_o} \qquad e_L = \frac{M_L}{F}$$

M_{eo}, M_e . . . The determinate prestressing moment equal to $F_o\ e$ or $F\ e$, respectively, depending on the actually acting prestressing force.

$\left.\begin{matrix}(M_{eo})_i \\ (M_e)_i\end{matrix}\right\}$ . . . The indeterminate prestressing moment due to prestressing and the continuity of the structure, depending upon the actually acting prestressing force.

e_{To}, e_T . . . Prestressing thrust eccentricity due to the prestressing force (before and after losses, respectively), the cable eccentricity and the continuity of the structure;

$$e_{To} = e + \frac{(M_{eo})_i}{F_o} \qquad e_T = e + \frac{(M_e)_i}{F}$$

When η is constant for all members,

$$e_{To} = e_T$$

e_{Ro}, e_R . . . Resultant thrust eccentricity after release and after deduction of all losses in the prestressing force respectively;

$$e_{Ro} = e_{To} + e_{Do} + e_{Lo}$$

$$e_R = e_T + e_D + e_L$$

f_{cp} . . . Permissible concrete compressive stress.

f_c^c . . . Concrete stress at c.g.s.

f_{bo}, f_b . . . Concrete stress at the bottom fibers after release and after deduction of all losses in the prestressing force, respectively.

f_{to}, f_t . . . Concrete stress at the top fibers after release and after deduction of all losses in the prestressing force, respectively.

e . . . Loss of steel stress due to elastic deformations of concrete upon transfer.

s . . . Loss of steel stress due to shrinkage of concrete.

p . . . Steel stress loss due to creep of concrete.

t . . . Steel stress loss due to creep of steel.

f_s . . . Total steel stress loss.

f_{so} . . . Prestress steel stress after release.

f_{si} . . . Prestress steel stress before release.

f_{se} . . . Prestress steel stress after all losses.

f_{sp} . . . Steel working stress.

f'_s . . . Ultimate strength of steel.

E_s . . . Modulus of elasticity of steel.

E_c . . . Modulus of elasticity of concrete.

III - PROPOSED DESIGN PROCEDURE

The analysis of prestressed concrete structures differs from that of ordinary reinforced concrete in that a more careful determination of the maximum and minimum moments is required. These values should be determined for sections along the entire length of the structural members rather than at a few selected "critical" sections. Furthermore, the analysis of indeterminate and determinate prestressed concrete structures differs in that the eccentricity of the prestressing thrust eccentricity in the former is generally displaced from the cable curve because of the continuity. That is, the fixed end moments due to prestressing are distributed through the structure. The following chronological design procedure is suggested, allowing for these differences:

(a) INFLUENCE LINES - Influence lines for bending moments at the likely critical sections (sections of maximum amplitude of moment variation) are determined for several ratios of member stiffnesses. In case of anticipated

uniform cross-sections, the stiffness factor is $4 \frac{I}{L}$.

When a variable cross-section member is desirable, the stiffness factor is again proportional to $\frac{I_c}{L}$ where I_c is the moment inertia at the section with the minimum depth in the member. The proportionality factor is a function of the assumed depth variation and may be calculated or obtained, for rectangular sections, from tables*.

(b) PRELIMINARY SELECTION OF STIFFNESS RATIOS - The influence line results allow calculation of the algebraic maximum and minimum live load moment values at the critical sections for the several stiffness ratios of step (a). These values are compared and stiffness ratios are assumed so that the live load moment variation for any critical section and for sections along each member is a minimum. The minimum live load variation at a section is desired since the concrete has zero allowable tensile stress, and the resultant thrust

*HANDBOOK OF FRAME CONSTANTS - Beam Factors and Moment Coefficients for Members of Variable Section - Portland Cement Ass'n, Chicago. (1947).

eccentricity must be within the kern of the section. The least moment variation (absolute value) along a member is desirable since this results in an economical uniform section for the member with more volume of the material subjected to higher stresses.

(c) FIRST DESIGN TRIAL -

(1) Maximum and minimum live load moments, based upon the assumed relative stiffnesses, are determined for a number of sections along the structural members. The number of the sections will depend upon the individual case and will usually vary from five to ten for each member;

(2) Neglecting the effect of dead load moments, calculate the required critical section moduli from Equation (1) in Appendix "A" and the computed live load moments.

(d) SECOND DESIGN TRIAL -

(1) Revise the live load moments to conform with the new stiffness ratios;

(2) Recalculate the required section moduli at the critical sections using Equation (1) of Appendix "A" and neglecting dead load effect;

(3) Calculate the dead load moments for areas corresponding to the above section moduli.

(e) THIRD DESIGN TRIAL - The proper equations in Appendix "A" are used to calculate the required section moduli. The procedure is repeated (this third trial usually suffices) until the sections on which the dead load moments and stiffness ratios are based are in agreement with the required sections.

(f) DETERMINATION OF CABLE ORDINATES -

(1) The final sections being known, the proper equations of Appendix "A" are used for the determination of the required prestressing force in each member and the required prestressing thrust eccentricity at each designed (critical) section, as well as the limits of the variation of the prestressing thrust eccentricity at the other sections selected in the "First Design Trial". Use equations (4) and (5) of Appendix "A" for these calculations. A diagram of the structure is then drawn, using the member centroidal axes. The limits of the prestressing thrust eccentricity are plotted for the above sections, through which a smooth curve is drawn. The prestressing thrust eccentricity is then evaluated graphically for the sections at which limits were set;

(2) The geometrical characteristics of the cable curve of each member are selected by visual inspection of the live load moment diagram. The cable curves are often assumed as parabolic of the second, third or fourth degree which are defined by one, two or three parameters respectively. Thus the equation of the cable curve for member I is:

$$e^{I} = f_{I}\,(x, e^{I1}, e^{I2}, e^{I3} \ldots\ldots e^{In})$$

where "x" is the independent variable along the member length and the parameters are the unknown cable ordinates at sections 1, 2, 3 .... n in member I. The number of parameters used depends upon the type of cable curve considered necessary. Long spans may necessitate assuming a powers or trigonometric series with n equal to five. The fixed-end moments due to prestress in member I are functions only of its cable curve parameters and may be expressed as follows:

$$(F.E.M.)_{F}^{IL} = F\,g_{I}\,(e^{I1}, e^{I2}, e^{I3} \ldots\ldots e^{In})$$

These fixed-end moments for the various members in the rigid structure are balanced by moment distri-

bution. A "Direct Method of Moment Distribution" if possible, is preferable.* The resulting balanced moments at the joints are (for example at the left terminal of member I):

$$(M_e^{IL})_i = q\ (F^I, e^{I1}, e^{I2} \ldots\ldots e^{In}, F^{II}, e^{II1}, e^{II2} \ldots\ldots e^{IIm}, F^{III}, e^{III1} \ldots\ldots)$$

In general, the indeterminate prestressing moment at any joint of the structure is a function of all cable curve parameters in the structure;

(3) The prestressing moment (for example, at any section in member I) is expressed in terms of the cable parameters by combining the determinate prestressing moment, M_e^{Ij}, at that section with the corresponding indeterminate prestressing moment, $(M_e^{Ij})_i$. Thus the prestressing thrust eccentricity, e_T^{Ij}, is obtained by dividing the section prestressing moment by the corresponding prestressing force. Or for section j in member I:

$$e_T^{Ij} = \frac{M_e^{Ij} + (M_e^{Ij})_i}{F^{Ij}} = e^{Ij} + \frac{(M_e^{Ij})_i}{F^{Ij}}$$

* "A DIRECT METHOD OF MOMENT DISTRIBUTION", A.S.C.E. Transactions, Vol. 102, p. 561 (1937).

(4) The expressions for the prestressing thrust eccentricities of step (3) are equated to those evaluated in step (1) for their respective sections. The resulting simultaneous equations are solved and the actual cable curve parameters are obtained. This allows plotting of the actual cable curves. The number of simultaneous equations equals the number of cable curve parameters for the structure. The form of the simultaneous equation is:

$$e_T^{Ij} = e^{Ij} + \frac{(M_e^{Ij})_i}{F^{Ij}}$$

$$e_T^{IIj} = e^{IIj} + \frac{(M_e^{IIj})_i}{F^{IIj}}$$

$$e_T^{IIIj} = e^{IIIj} + \frac{(M_e^{IIIj})_i}{F^{IIIj}}$$

The number of equations may be large, depending upon the type and dimensions of the structure. However, the solutions may be readily obtained by successive approximations, as discussed in Appendix "B".

(g) CHECK -

(1) The check is that the resultant thrust eccentricities must at all times be within the kern limits of any section for zero tensile stress;

(2) The assumed value of η is checked by obtaining an average concrete stress along the cable centroid and computing the shrinkage of the concrete and the creep of the concrete and steel indicating the resulting loss of steel strain. Recommendations are available for the shrinkage and creep of concrete.*

IV - PRESTRESSED MONOLITHIC FRAME DESIGN

A design load of 17.5 kips was selected to initiate the frame design with a working stress of 0.45×5000 psi = 2250 psi. The derived equations, used for the design, are based upon zero flexural tensile stresses. In addition, steel strains of approximately 4×10^{-3} inch per inch for post-tensioning at a concrete age of three weeks leads to an assumption of $\eta = 0.85$ for this design.

Steps (a) and (b) of the proposed design procedure are similar to the familiar methods used in the design of ordinary reinforced concrete continuous structures

* FIRST REPORT ON PRESTRESSED CONCRETE - Institution of Structural Engineers. London, Sept. 1951.

and will therefore only be briefly outlined in this presentation.* The magnitude of the bending moments at the critical sections III1, III5, II3, II1, I5 and I1, as shown in Fig. 1, was determined for various combinations of the stiffness ratios $\frac{I^{III}}{I^{I}}$ and $\frac{I^{III}}{I^{I}} \frac{8}{9}$.

The moments for the above section were plotted versus $\frac{I^{III}}{I^{I}}$ for several values of $\frac{8}{9} \frac{I^{III}}{I^{II}}$. The stiffness ratios $\frac{I^{III}}{I^{I}} = 1.00$, and $\frac{8}{9} \frac{I^{III}}{I^{II}} = 0.89$ were adjudged to provide the least moment variation for the above sections and hence were selected to initiate the first design trial.

FIRST DESIGN TRIAL - The required section moduli for the above sections were computed by use of Equation 1, Appendix A, neglecting the dead load moments. The computations for M_L moments in this connection were based upon the selected stiffness ratics with the 17.5 kip load and reactions as placed in Figs. 2, 3, 4 and 5. Rectangular cross-sections

*CONTINUOUS CONCRETE BRIDGES - Second Edition - Portland Cement Ass'n, Chicago, Ill.

were chosen for the frame members in order to reduce pouring and instrumentation difficulties. The section moments of inertia were then computed, followed by the computation of the stiffness ratios. The new stiffness ratio $\frac{I^{II}}{I}$ was found to be equal to 0.64, while the stiffness ratio $\frac{I^{II}}{I}\ \frac{8}{9}$ remained equal to 0.89. The computed section moduli required the following rectangular cross-sections:

Member	Critical Section	Required Section Modulus - in^3		Width in.	Depth in.
I	1	$Z_b^{I1} = Z_t^{I1} = Z^{I1} = 292$		9	14
II	1	$Z^{II1} = 139$	Use	9	12
III	1	$Z^{III1} = 214$		9	12

The leg members were taken as 9 by 12 rather than 9 by 10 since a decrease in $\frac{I^{III}}{I^{II}}\ \frac{8}{9}$ leads to greater moment variation in member II. Table I was then prepared with the new stiffness ratio and selected sections.

SECOND DESIGN TRIAL - The constants k_D, k_L and α were computed for the sections of Fig. 1 using the values of Table I. These constants with the value of $Z_r = 1$ then determine the correct equations

of Appendix A to be used in the further computation of section moduli. The significant results were:

Member	Critical Section	Required Section Modulus	Width	Depth
I	1	303	9.25	14
III	1	194	9.25	12
II	1	152	9.25	12

The original dead load moments of Table I were changed after the "Second Trial" to correspond to a width increase in all members of 0.25 inches. The new values are shown in Table I with an asterisk. The moment diagrams are illustrated in Figs. 6 through 10. The "Third Trial" design was then performed using the new section dead load moments with the same live load moments (no change in stiffness ratios).

TABLE I

DEAD LOAD MOMENTS FOR TRIALS 2 and 3 - Inch kips

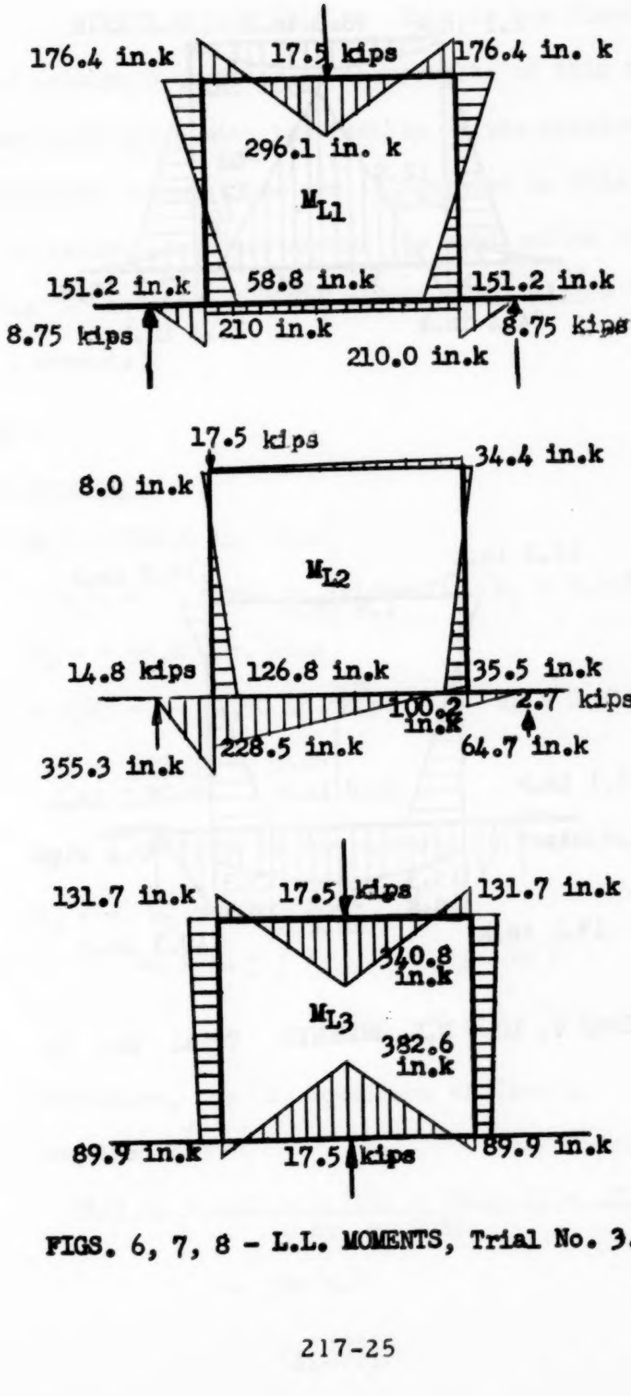
Member	Section	M_{D1}	M_{D2}
I	1	- 31.5 *	+ 98.8 *
		- 30.7	+ 96.1
	2	- 30.5 *	+ 65.0 *
		- 29.7	+ 63.2
	3	- 27.4 *	+ 33.0 *
		- 26.7	+ 32.2
	4	- 22.5 *	+ 3.3 *
		- 21.9	+ 3.2
	5	- 15.3 *	- 24.4 *
		- 14.8	- 23.8
	6	- 49.1 *	+ 12.9 *
		- 47.8	+ 12.6
	7	- 23.7 *	+ 7.3 *
		- 23.1	+ 7.1
II	1	- 12.2 *	+ 1.1 *
		- 11.8	+ 1.1
	2	+ 10.8 *	- 18.1 *
		+ 10.6	- 17.6
	3	+ 33.8 *	- 37.3 *
		+ 32.9	- 36.3
III	1	+ 1.8 *	+ 15.1 *
		+ 1.7	+ 14.6
	2	+ 0.9 *	+ 14.1 *
		+ 0.9	+ 13.8
	3	- 1.8 *	+ 11.4 *
		- 1.8	+ 11.1
	4	- 6.2 *	+ 7.1 *
		- 6.0	+ 6.9
	5	- 12.1 *	+ 1.1 *
		- 11.8	+ 1.1

* Figures with asterisk show revised D.L. Moments for Sections 9-1/4" wide.

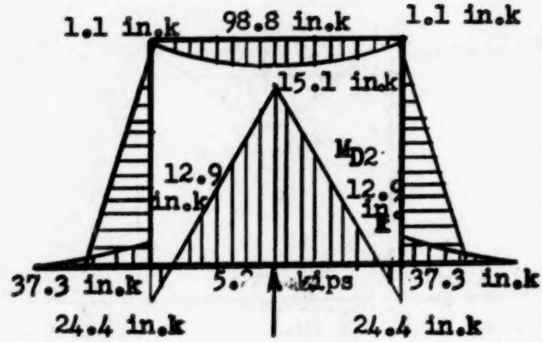
TABLE I (continued)

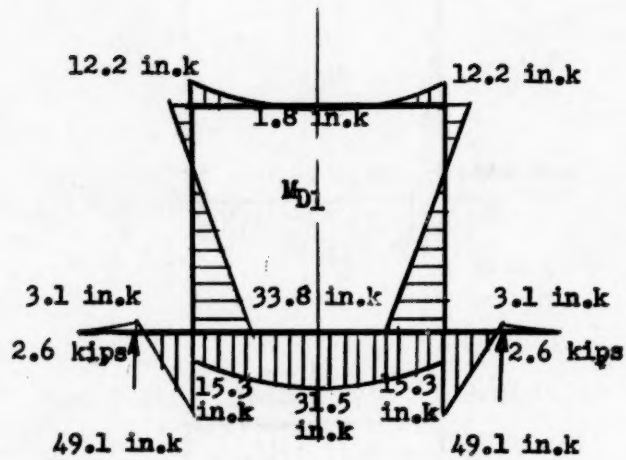
LIVE LOAD MOMENTS FOR TRIALS 2 and 3 - Inch kips

Member	Section	M_{L1}	M_{L2}	M'_{L2}	M_{L3}
I	1	- 58.8	- 96.5	- 96.5	+ 382.6
	2	- 58.8	- 129.5	- 63.5	+ 264.6
	3	- 58.8	- 162.5	- 30.5	+ 146.4
	4	- 58.8	- 189.8	+ 2.5	+ 28.3
	5	- 58.8	- 228.5	+ 35.5	- 89.9
	6	- 210.0	- 355.3	- 64.7	0
	7	- 105.0	- 177.7	- 32.3	0
II	1	- 176.4	- 8.0	- 34.4	- 131.7
	2	- 12.6	+ 59.4	+ 33.2	- 110.9
	3	+ 151.2	+ 126.8	+ 100.2	- 89.9
III	1	+ 296.1	- 21.2	- 21.2	+ 340.8
	2	+ 178.0	- 17.9	- 24.5	+ 222.8
	3	+ 59.8	- 14.6	- 27.8	+ 104.6
	4	- 58.3	- 11.3	- 31.1	- 13.4
	5	- 176.4	- 8.0	- 34.4	- 131.7



FIGS. 6, 7, 8 - L.L. MOMENTS, Trial No. 3.





FIGS. 9, 10 - D.L. MOMENTS, Trial No. 3.

THIRD DESIGN TRIAL - This is the final design for the section properties if the results of this trial substantially agree with the results of the previous trial. The numerical computations are illustrated in this design trial to assist in understanding the application of the Appendix "A" equations. The computations follow (use Table I moments):

MEMBER I

(a) Section 1

$M_L = + 382.6$ in. kips

$- k_L M_L = - 96.50$ in. kips $\qquad k_L = 0.252$

$M_D = + 98.80$ in. kips

$- k_D M_D = - 31.50$ in. kips $\qquad k_D = 0.319$

$$\alpha = \frac{382.62}{98.80} = 3.88$$

$Z_r = 1$ for the chosen symmetrical section.

$$\left.\begin{array}{r} k_D + \alpha k_L > 0 \\ \alpha > -1 \end{array}\right\} \text{and} \; (Z_r - \eta)\,(1 + \alpha) + (k_D + \alpha k_L)\,(\eta Z_r - 1) > 0$$

Therefore, use (b) equations of Case 1.

Required Z^{I1} =

$$\frac{98.8\,(1 + 0.85 \times 0.319) + 382.6\,(1 + .85 \times .252)}{0.85 \times 2.25} =$$

$$= 308 \text{ in}^3$$

The section of Trial Design No. 2, $b = 9.25$ in. and $h = 14.0$ in., with $Z^I = 303\ in^3$, is adopted for member I.

From equation (3), Appendix "A":

$$F_o^I = \frac{129.5\ [98.8\ (1.319) + 382.6\ (1.252)]}{(.85)\ (612)} =$$

$$= 153 \text{ kips}$$

From equation (4) the lower limit of e_T^{I1} is:

$$e_T^{I1} = -\frac{382.6 + 98.8}{.85 \times 153} + \frac{303}{129.5} = -1.38 \text{ in.}$$

The upper limit coincides with the lower limit of -1.38 in., that is, e_T is uniquely determined at sections when the required F_o, based upon the section, is used.

(b) Section 5

$F_o^{I5} = F_o^{I1} = 153$ kips; the prestressing force is assumed constant along the member.*

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* The prestressing force does not necessarily need to be assumed constant. If there is a variation of prestressing force due to frictional losses, the prestressing thrust eccentricity limits may be fixed by the force at its section. However, the required prestressing force must be developed at the critical sections, such as I1.

$M_L = -228.5$ in. kips

$-k_L M_L = +35.5$ in. kips $\quad k_L = 0.155$

$M_D = -15.2$ in. kips

$-k_D M_D = -24.4$ in. kips $\quad k_D = -1.61$

Section required at 1 is adequate for moments of Section 5.

From equations (4) and (5), Appendix "A", the limits for e_T^{I5} are:

$$\text{Upper limit} = \frac{+15.2}{(.85)} + \frac{228.5}{(153)} - 2.34\,^{*} = -0.47 \text{ in.}$$

$$\text{Lower limit} = \frac{+24.4}{(.85)} - \frac{35.5}{(153)} + 2.34 = +2.24 \text{ in.}$$

(c) Section 6

$F_o^{I6} = F_o^{I1}$

$M_L = -355.3$ in. kips

$M_D = -49.10$ in. kips

$k_L M_L = 0$

$-k_D M_D = +12.9$ in. kips

---

* When $M_L < 0$ $(-c_t)$ and c_b are interchanged in equations (4) and (5).

Section required at 1 is adequate for moments of section 6.

From equations (4) and (5), Appendix "A", the limits for e_T^{I6} are:

$$\text{Upper limit} = + \frac{355.3 + 49.1}{130} - \frac{302}{129.5} =$$

$$3.11 - 2.33 = + 0.77 \text{ in.}$$

$$\text{Lower limit} = - \frac{12.9}{130} + \frac{302}{129.5} =$$

$$- .10 + 2.34 = + 2.24 \text{ in.}$$

The prestressing thrust eccentricities at sections 5 and 6 of member I must be equal, hence:

$$+ .77 \text{ in.} \leq e_T^{I6} = e_T^{I5} \leq + 2.24 \text{ in.}$$

<u>MEMBER III</u>

<u>(a) Section 1</u>

$M_L = + 340.8$ in. kips

$- k_L M_L = - 21.2$ in. kips $\qquad k_L = .062$

$M_D = + 15.0$ in. kips

$- k_D M_D = - 1.79$ in. kips $\qquad k_D = - .12$

$\alpha = 22.6 \qquad k_D + \alpha k_L > 0$ and $\alpha > -1$

$$\left.\begin{array}{r} k_D + \alpha k_L > 0 \\ \alpha > -1 \end{array}\right\} \text{ and } (Z_r - \eta)(1 + \alpha) + (k_D + \alpha k_L)(\eta Z_r - 1) > 0 \;;$$

Therefore, use Case I (b) equations.

From equations (6) and (7) of Case I (b) in Appendix A:

$$Z^{III1} = \frac{15.0\ (1 - .85 \times .12) + 340.8\ (1 + .85 \times .062)}{1.92}$$

$$= 196 \text{ in}^3$$

Use 9.25 in. x 12 in. with $Z^{III1} = 222 \text{ in}^3$

From equation (3) Appendix A:

$$F_o^{III1} = \frac{111\ [15\ (1 - .12) + 340.8\ (1.062)]}{(.85)\ (444)} = 111 \text{ kips}$$

From equations (4) and (5) in Appendix A, the limits of e_T^{III1} are:

$$\text{Upper limit} = -\frac{15 + 340.8}{(.85)\ (111)} + 2 = -3.78 + 2.00$$

$$= -1.78 \text{ in.}$$

$$\text{Lower limit (check)} = +\frac{21.2 - 1.79}{94} - 2.00$$

$$= +0.21 - 2.00 = -1.79 \text{ in.}$$

Again the upper and lower limits nearly coincide, since the section modulus used is near that required.

(b) Section 5

$M_L = -176.4$ in. kips $\qquad k_L = 0$

$M_D = -12.2$

$-k_D M_D = +1.1 \qquad k_D = .09$

The limits of the prestressing thrust eccentricity are obtained by use of equations (4) and (5) of Appendix A. They are, for e_T^{III5} :

$$\text{Lower limit} = +\frac{176.4 + 12.2}{94} - \frac{222}{111} = 0$$

$$\text{Upper limit} = -\frac{1.1}{94} + 2.00 = +1.99 \text{ in.}$$

$$\text{Hence } 0 \leqslant e_T^{III5} \leqslant +1.99$$

MEMBER II

(a) Section 3

$M_L = +151.2$ in. kips

$M_D = +33.8$ in. kips

$-k_L M_L = -89.9$ in. kips $\qquad k_L = 0.594$

$-k_D M_D = -37.3$ in. kips $\qquad k_D = 1.10$

$\alpha = 4.47$

$$\left.\begin{array}{l} k_D + \alpha k_L > 0 \\ \alpha > -1 \end{array}\right\} \text{ and } (Z_r - \eta)(1 + \alpha) + (k_D + \alpha k_L)\cdot(\eta Z_r - 1) > 0$$

Hence use Case I (b) equations.

Required Z^{II3} =

$$\frac{33.8\ (1 + .85 \times 1.10) + 151.2\ (1 + .85 \times .594)}{1.92} =$$

$$= 153\ \text{in}^3$$

Use same section as Member III, $Z^{III} = 222\ \text{in}^3$

From equation (3):

$$F_o^{III3} = \frac{111\ [33.8\ (2.1) + 151.2\ (1.59)]}{0.85\ \ (444)} = 92\ \text{kips}$$

Hence, from equations (4) and (5), the limits of e_T^{III3} are:

$$\left.\begin{array}{l}\text{Outer or}\\ \text{Upper limit}\end{array}\right\} = -\frac{33.8 + 151.2}{92 \times 0.85} + \frac{222}{111}$$

$$= -2.37 + 2.00 = -0.37\ in$$

$$\left.\begin{array}{l}\text{Inner or}\\ \text{Lower limit}\end{array}\right\} \begin{array}{l}\text{(check)}\\ = \dfrac{37.3 + 89.88}{92 \times 0.85} - 2.00\end{array}$$

$$= 1.63 - 2.00 = -0.37\ in$$

(b) Section 1

$M_L = -176.4$ in. kips

$M_D = -12.1$ in. kips

$k_L M_L = 0$ $\qquad$ $k_L = 0$

$$-k_D M_D = +1.08 \qquad k_D = 0.09$$

$$F_o^{III} = 92 \text{ kips} \qquad F^{III} = 78.2 \text{ kips}$$

From equations (4) and (5) of Appendix A, the e_T^{III} limits are:

$$\left.\begin{array}{c}\text{Upper}\\ \text{or}\\ \text{Outer}\end{array}\right) = + \frac{12.1 + 176.4}{78.2} - \frac{222}{111} = +0.41 \text{ in.}$$

$$\left.\begin{array}{c}\text{Lower}\\ \text{or}\\ \text{Inner}\end{array}\right) = - \frac{1.08}{78.2} + 2.00 = +2.00 \text{ in.}$$

$$+0.41 \leq e_T^{III} \leq +2.00$$

This completes the determination of the section moduli, prestressing forces and limits of prestressing thrust eccentricities. The remaining problem is that of calculating the exact cable location compatible with the computed prestressing thrust eccentricities.

Determination of Cable Ordinates

The prestressing moments approximately follow the shape of the cables, and since prestressing moments must counteract the live and dead load moments as far as possible, the moment diagrams in Figures 6 through 10 indicate that the prestressing cables should have such shapes as shown in the following figure:

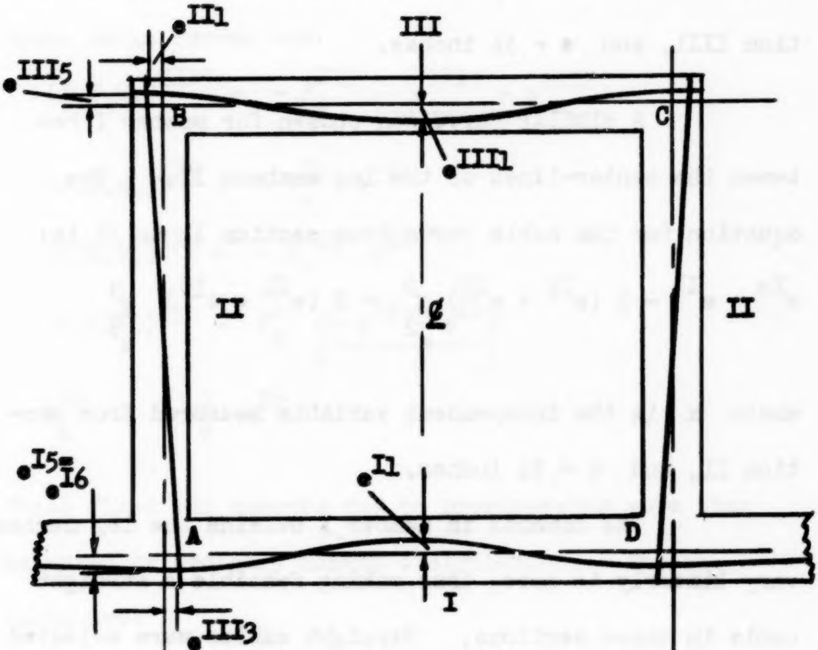


FIGURE 11

A third degree parabola symmetrical about the center-line was chosen to represent the cable curve in member III. The equation for the cable curve from section III1 to III5 is:

$$e^{IIIx} = - e^{III1} + 3 \left(e^{III1} + e^{III5}\right) \frac{x^2}{a^2}$$

$$- 2 \left(e^{III1} + e^{III5}\right) \frac{x^3}{a^3}$$

where x is the independent variable measured from section III1, and $a = 54$ inches.

A similar curve was chosen for member I between the center-lines of the leg members II. The equation for the cable curve from section I1 to I5 is:

$$e^{Ix} = e^{I1} - 3 \left(e^{I1} + e^{I5}\right) \frac{x^2}{a^2} + 2 \left(e^{I1} + e^{I5}\right) \frac{x^3}{a^3}$$

where x is the independent variable measured from section I1, and $a = 54$ inches.

The moments in member I outside the leg members vary linearly to zero, thus making feasible a straight cable in these sections. Straight cables were selected for the legs (member II) since all moments applied to these members vary linearly. Thus the diagram of Fig. 1

represents the cable locations with the chosen cable parameters e^{I1}, e^{I5}, e^{III1}, e^{III5}, e^{II1} and e^{II3}. The fixed end moments for the members were then expressed as functions of these parameters, assuming the prestressing force constant throughout the member. The fixed end moment sign convention, for these equations, is that clockwise moments acting upon joints are positive. The parameters, cable ordinates, are positive as illustrated in Fig. 11. The fixed end moments for these cable curves are:

$$(F.E.M.)^{III5} = -F_o^{III}\left(\frac{-e^{III5} + e^{III1}}{2}\right)$$

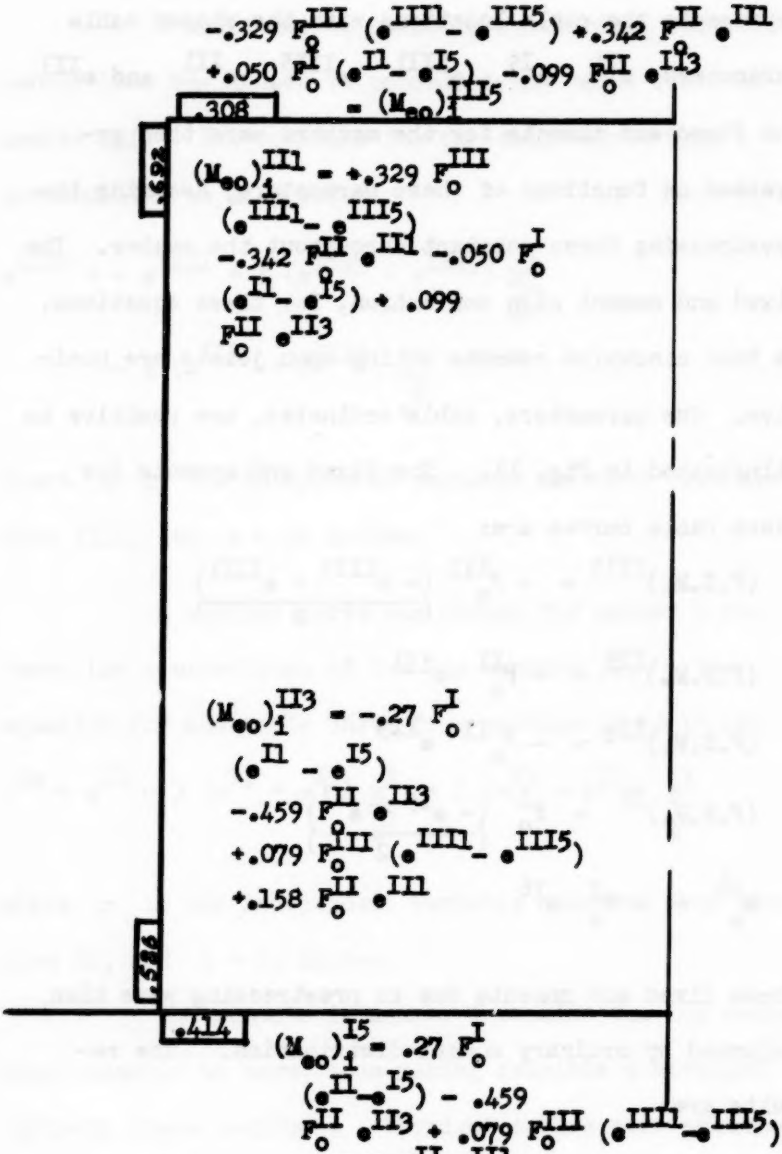
$$(F.E.M.)^{II1} = -F_o^{II}\, e^{II1}$$

$$(F.E.M.)^{II3} = -F_o^{II}\, e^{II3}$$

$$(F.E.M.)^{I5} = F_o^{I}\left(\frac{-e^{I5} + e^{I1}}{2}\right)$$

$$M_e^{I6} = -F_o^{I}\, e^{I6}$$

These fixed end moments due to prestressing were then balanced by ordinary moment distribution. The results are:



BALANCED END MOMENTS DUE TO PRESTRESSING

The computation of the prestressing moment at any section was then possible. The prestressing moments at the sections considered in the design were then computed. Moments producing tension on the inside fibers of the frame or in the bottom fibers of the cantilever are now considered positive. The equations are:

$$M_{eo}^{I6} = - F_o^{I} \, e^{I5}$$

$$M_{eo}^{II3} = - .27 \, F_o^{I} (e^{I5} - e^{I1}) - .54 \, F_o^{II} \, e^{II3}$$
$$- .16 \, F_o^{II} \, e^{II1} - .079 \, F_o^{III} (- e^{III5} + e^{III1})$$

$$M_{eo}^{I1} = F_o^{I} (- .27 \, e^{I5} - .73 \, e^{I1}) - .079 \, F_o^{III}$$
$$(- e^{III5} + e^{III1}) - .158 \, F_o^{II} \, e^{II1}$$
$$+ .46 \, F_o^{II} \, e^{II3}$$

$$M_{eo}^{III1} = .66 \, F_o^{II} \, e^{II1} + .329 \, F_o^{III} (- e^{III5} + e^{III1})$$
$$- .05 \, F_o^{I} (- e^{I5} + e^{I1}) + .099 \, F_o^{II} \, e^{II3}$$

$$M_{eo}^{III5} = F_o^{III} (.67 \, e^{III5} + .33 \, e^{III1}) - .050 \, F_o^{I}$$
$$(- e^{I5} + e^{I1}) - .34 \, F_o^{II} \, e^{II1} + .10 \, F_o^{II} \, e^{II3}$$

$$M_{eo}^{III1} = - F_o^{III} (.33\ e^{III5} + .67\ e^{III1}) - .050\ F_o^{I}$$
$$(- e^{I5} + e^{I1}) - .34\ F_o^{II}\ e^{II1}$$
$$+ .10\ F_o^{II}\ e^{II3}$$

When the ratio $\eta = \frac{F}{F_o}$ is considered constant for all members of the structure, the prestressing moments M_e are equal to $\eta\ M_{eo}$. In this case, the prestressing thrust eccentricities are identical before and after the losses in prestressing forces. This assumption of constant η was made for the frame under consideration. The equations for the prestressing moments when divided by the corresponding prestressing forces determined the prestressing thrust eccentricities at the respective sections. This operation, after substitution of the numerical values of the prestressing forces, results in the following equations:

$$e_T^{II3} = - 0.45\ e^{I5} + 0.45\ e^{I1} - 0.54\ e^{II3}$$
$$- 0.16\ e^{II1} + 0.10\ e^{III5} - 0.10\ e^{III1}$$

$$e_T^{II1} = + 0.083\ e^{I5} - 0.083\ e^{I1} + 0.099\ e^{II3}$$
$$+ 0.66\ e^{II1} - 0.40\ e^{III5} + 0.40\ e^{III1}$$

$$e_T^{III5} = + 0.069\, e^{I5} - 0.069\, e^{I5} + 0.083\, e^{II3}$$
$$- 0.28\, e^{II1} + 0.67\, e^{III5} + 0.33\, e^{III1}$$

$$e_T^{III1} = + 0.069\, e^{I5} - 0.069\, e^{I1} - 0.083\, e^{II3}$$
$$- 0.28\, e^{II1} - 0.33\, e^{III5} - 0.67\, e^{III1}$$

$$e_T^{I1} = - 0.27\, e^{I5} - 0.73\, e^{I1} + 0.276\, e^{II3}$$
$$- 0.096\, e^{II1} + 0.057\, e^{III5} - 0.057\, e^{III1}$$

$e_T^{I5} = e^{I5}$ for the cantilever.

In the above equations, the influence of the cable ordinate at any of the designed sections upon the thrust eccentricity of other designed sections is established.

The design requirements as previously established are:

$$e_T^{II3} = - 0.37$$
$$0.41 \leq e_T^{II1} \leq 2.00$$
$$0 \leq e_T^{III5} \leq 1.99$$
$$e_T^{III1} = - 1.78$$
$$e_T^{I1} = - 1.38$$
$$+ 0.77 \leq e_T^{I5} \leq + 2.24$$

In order to facilitate the determination of the required cable ordinates, the values of e_T^{II1}, e_T^{III5} and e_T^{I5} were selected and used with the unique values at the other sections. Hence:

$$e_T^{II3} = -0.45\,e^{I5} + 0.45\,e^{I1} - 0.54\,e^{II3} - 0.16\,e^{II1} + 0.10\,e^{III5} - 0.10\,e^{III1} = -0.37$$

$$e_T^{II1} = +0.08\,e^{I5} - 0.08\,e^{I1} + 0.10\,e^{II3} + 0.66\,e^{II1} - 0.40\,e^{III5} + 0.40\,e^{III1} = +1.10$$

$$e_T^{III5} = +0.07\,e^{I5} - 0.07\,e^{I1} + 0.08\,e^{II3} - 0.28\,e^{II1} + 0.67\,e^{III5} + 0.33^{III1} = 0.92$$

$$e_T^{III1} = +0.07\,e^{I5} - 0.07\,e^{I1} - 0.08\,e^{II3} - 0.28\,e^{II1} - 0.33^{III5} - 0.67\,e^{III1} = -1.78$$

$$e_T^{I1} = -0.27\,e^{I5} - 0.73\,e^{I1} + 0.28\,e^{II3} - 0.10\,e^{II1} + 0.06\,e^{III5} - 0.06\,e^{III1} = -1.38$$

$$e_T^{I5} = e^{I5} = 1.35$$

Successive approximations were used to obtain a solution to this set of equations. The results were:

$$e^{II3} = -0.13 \text{ in.}$$

$$e^{II1} = 0$$

$$e^{III5} = 0$$

$$e^{III1} = -2.70 \text{ in.}$$

$$e^{I1} = -1.24 \text{ in.}$$

$$e^{I5} = +1.35 \text{ in.}$$

It should again be noted that the cable ordinate signs, as indicated above, are positive when the prestressing force produces tension on the inside fibers. These ordinates are substituted as positive values in the cable curve equations since the results are in agreement with the original sketch of the cable locations of Fig. 11. The values of the cable ordinates and the prestressing thrust eccentricities are illustrated in Table II.

TABLE II

CABLE ORDINATES AND PRESTRESSING THRUST ECCENTRICITY

Member	Section	Cable ordinates e from cable Curve Equations	$(M_e)_{i/F}$	e (Corrected Signs)	e_T
I	1	+ 1.24	- .15	- 1.24	- 1.39
	2	+ 0.84	- .15	- .84	- .99
	3	- .12	- .15	+ .12	- .03
	4	- .94	- .15	+ .94	+ .79
	5	- 1.36	- .15	+ 1.36	+ 1.21
	6	- 1.36	0	+ 1.36	+ 1.36
	7	- 1.02	0	+ 1.02	+ 1.02
II	1	0	+ 1.10	0	+ 1.10
	2	+ .06	+ .42	- .06	+ .36
	3	+ .13	- .25	- .13	- .38
III	1	- 2.70	+ .92	- 2.70	- 1.79
	2	- 2.28	+ .92	- 2.28	- 1.37
	3	- 1.31	+ .92	- 1.31	- .40
	4	- .43	+ .92	- .43	+ .49
	5	0	+ .92	0	+ .92

Check

The resultant thrust eccentricities before and after losses (creep and shrinkage) were readily determined and are listed in Tables III and IV.

The check upon the assumed loss of prestressing force may be accomplished with the following equation (for example, member I):

$$1 - \eta = \frac{F_o^I - F^I}{F_o^I} = \frac{E_s C_c}{f_s} \left(\frac{F_o^I e_{Ro}^{Ij}}{I^{Ij}} e + \frac{F_o^I}{A^{Ij}} \right)_{aver.} + E_s f_s C_s + S_c \frac{E_s}{f_s}$$

where S_c = shrinkage of concrete in inches/inch

C_c = creep of concrete per psi stress-in/in.

C_s = creep of steel per psi stress -in/in.

This check was performed for the various members for all possible loadings and an average stress in the concrete adjacent to the steel f_c^c was computed. This highest average f_c^c for the various loadings was used in the calculations. The steel considered for the post-tensioning of the frame exhibited no creep at 120,000

psi, $\Delta_s = 0$, and E_s of 30×10^6 psi. Linear creep and shrinkage factors were assumed in accordance with those recommended in the "First Report on Prestressed Concrete".* The results of these calculations are:

Member I

$$1 - \eta = \frac{30 \times 10^6}{120{,}000} \left(.3 \times 10^{-6} \, (1260)_{\text{aver.}} + .0002 \right)$$

$= .145 \approx 15\%$, which agrees with the selected value.

* "FIRST REPORT ON PRESTRESSED CONCRETE" - Institution of Structural Engineers - p.12. London. Sept. 1951.

Member II

$$1 - \eta = \frac{30 \times 10^6}{120{,}000}$$

$$[\ .3 \times 10^{-6}\ (850)_{aver.} + .0002\ ]$$

$$= .115 \approx 12\%,$$ or less than the assumed.

This is to be expected since the legs were over-designed, increasing the depth from 10.5 to 12 inches.

Member III

$$1 - \eta = \frac{30 \times 10^6}{120{,}000}$$

$$[\ .3 \times 10^{-6}\ (1160) + .0002]$$

$$= .138 \approx 14\%,$$ which again approximately agrees with the selected value.

TABLE III

RESULTANT THRUST ECCENTRICITIES BEFORE LOSSES

Member	Section	$e_{R_o} = \frac{M_{D1}+M_{L1}}{F_o} + e_T$	$e_{R_o} = \frac{M_{D1}+M_{L2}}{F_o} + e_T$	$e_{R_o} = \frac{M_{D1}+M'_{L2}}{F_o} + e_T$	$e_{R_o} = \frac{M_{D2}+M_{L3}}{F_o} + e_T$	Kern Limits
I	1	- 1.98	- 2.23	- 2.23	+ 1.77	± 2.33
	2	- 1.57	- 2.04	- 1.61	+ 1.17	"
	3	- .60	- 1.28	- .41	+ 1.14	"
	4	+ .26	- .60	+ .66	+ 1.00	"
	5	+ .73	- .39	+ 1.34	+ .46	"
	6	- .34	- 1.28	+ .61	+ 1.28	"
	7	+ .19	- .30	+ .65	+ .97	"
II	1	- .95	+ .88	+ .59	- .32	± 2.00
	2	+ .34	+ 1.12	+ .84	- 1.04	"
	3	+ 1.63	+ 1.37	+ 1.09	- 1.76	"
III	1	+ .91	- 1.98	- 1.97	+ 1.43	"
	2	+ .23	- 1.54	- 1.58	+ .78	"
	3	+ .12	- .55	- .64	+ .65	"
	4	- .10	+ .32	+ .15	+ .43	"
	5	- .80	+ .74	+ .49	- .27	"

TABLE IV

RESULTANT THRUST ECCENTRICITIES AFTER LOSSES
WITH η = 0.85

Member	Section	$e_R = \frac{M_{D1}+M_{L1}}{F} + e_T$	$e_R = \frac{M_{D1}+M_{L2}}{F} + e_T$	$e_R = \frac{M_{D1}+M_{L2}}{F} + e_T$	$e_R = \frac{M_{D2}+M_{L3}}{F} + e_T$	Kern Limits
I	1	- 2.07	- 2.36	- 2.36	+ 2.32	± 2.33
	2	- 1.67	- 2.22	- 1.71	+ 1.55	"
	3	- .69	- 1.49	- .47	+ 1.34	"
	4	+ .17	- .84	+ .64	+ 1.03	"
	5	+ .64	- .67	+ 1.36	+ .33	"
	6	- .63	- 1.74	+ .48	+ 1.26	"
	7	+ .05	- .53	+ .59	+ .97	"
II	1	- 1.31	+ .85	+ .51	- .57	± 2.00
	2	+ .06	+ .98	+ .63	- 1.29	"
	3	+ 1.12	+ .81	+ .49	- 2.00	"
III	1	+ 1.39	- 2.00	- 2.00	+ 2.00	"
	2	+ .53	- 1.55	- 1.66	+ 1.16	"
	3	+ .21	- .58	- .72	+ .83	"
	4	- .20	+ .30	+ .09	+ .42	"
	5	- 1.09	+ .71	+ .43	- .47	"

This completes the design of the monolithic frame, post-tensioned. It may be noted that the shear stresses when combined with the compressive stresses resulted in tensile stresses of less than 80 psi. Thus no stirrups were used. This calculation of maximum tensile stress is approximate, since there is no exact method of determining the stress distribution at the corners. It may also be noted that the computation of the end block stresses is omitted. Again the stress distribution is in question, and field experience with different tensioning methods and end blocks vary the solutions to this problem.

No allowances for frictional or transfer losses of steel stress were made in the design procedure. These problems have solutions in field technique. For example, the steel for post-tensioning the frame was initially over-stressed approximately Ten Per Cent to compensate for transfer and frictional losses. For long span structures, sliding bearing

blocks may be inserted at strategic points to alleviate frictional losses.*

CONCLUSIONS

The design procedure presented is relatively simple. The assumed stress conditions and resulting equations considerably reduce the labor of design. The equations are based upon zero allowable flexural tensile stresses; however, it has been observed that considerable economy may be effected by allowing tensile stresses of small magnitude. Equations allowing such stresses have been developed since the monolithic frame design.

Ultimate loads are well defined for determinate prestressed concrete structures** and may be used for defining section geometry and safety factors. This is not the case, however, for indeterminate prestressed concrete structures. Thus the ultimate loads of the frame could not be computed for this frame with a reasonable degree of accuracy.

*LEONHARDT, Fritz - "Continuous Prestressed Concrete Beams". Jour. A.C.I. March 1953.

**ABELES, P.W. - "The Use of High Strength Steel in Ordinary Reinforced and Prestressed Concrete Beams". Fourth Congress-Internat'l Ass'n for Bridge & Structural Engineering. 1952.

The monolithic frame design in this paper does not represent a feasible application of prestressed concrete. However, this frame represented an experimental construction for test purposes, and as such well suited the original purposes. It does illustrate, in conjunction with the equations, the effect of reverse moments at one section of a member. The frame, when compared to ordinary concrete construction, represents a considerable saving of steel and concrete. However, it is doubtful that a structure of this type could economically compete with ordinary reinforced concrete construction. Thus it appears improbable that this method of construction would be adopted for rigid frames of multi-story buildings. This is not the situation for bridges or such structures of long spans with little or no reversing of moments at any one section.

It is of interest to mention certain results of the monolithic frame tests in connection with conclusions regarding the design assumptions. The first is that the experimentally determined prestressing and live load moment distributions agreed extremely well with the calculated distributions. This agreement continued even with

cracked sections in the frame - that is, the ratio of stiffnesses remained approximately constant up to a 50 kip load. The ultimate load of the frame was 58 kips or 3.3 times the design live load for M_{L2} loading. The assumption of zero tensile stress and a compressive stress of $0.45\ f'_c$ appeared to lead to an adequate safety factor.

The assumption of η equal to 0.85 was experimentally verified for members I and III. Creep measurements were made over a period of three months and the losses were approximately Ten to Thirteen Per Cent. Since most of the creep takes place within the first two weeks of poststress, these results tend to corroborate the design assumptions.

No frictional losses were apparent in members II and III, while some loss was apparent at section 1 of member I.

As mentioned, no steel was used to reinforce the frame for tensile stresses induced through shear, and no "diagonal tension" failure occurred when the frame was tested.

In general, these conclusions regarding the monolithic frame construction and tests confirm the design assumptions. In addition, the frame design illustrates the suggested procedure as being a relatively rapid method of designing indeterminate prestressed concrete structures.

APPENDIX A

DESIGN EQUATIONS

These equations are presented as the final results as derived from equilibrium and the illustrated stress conditions. The equations from one of four cases are used to determine the values of Z_b, Z_t, F_o and e_T. The case to be used is uniquely determined from relationships involving the ratios of moments k_D*, k_L** and α. Within each case are two sets of equations whose use depends upon the additional ratio of section moduli, Z_r. The section moduli ratio may be assumed and the assumption checked with calculations. In many instances the ratio of section moduli is known and the values of Z_b, Z_t, F_o and e_T are then uniquely determined consistent with stress conditions of Figure 12. The equations follow:

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* $k_D = -1$ in all cases when M_D does not change.

** $k_L = 0$ in all cases when M_L does not change sign.

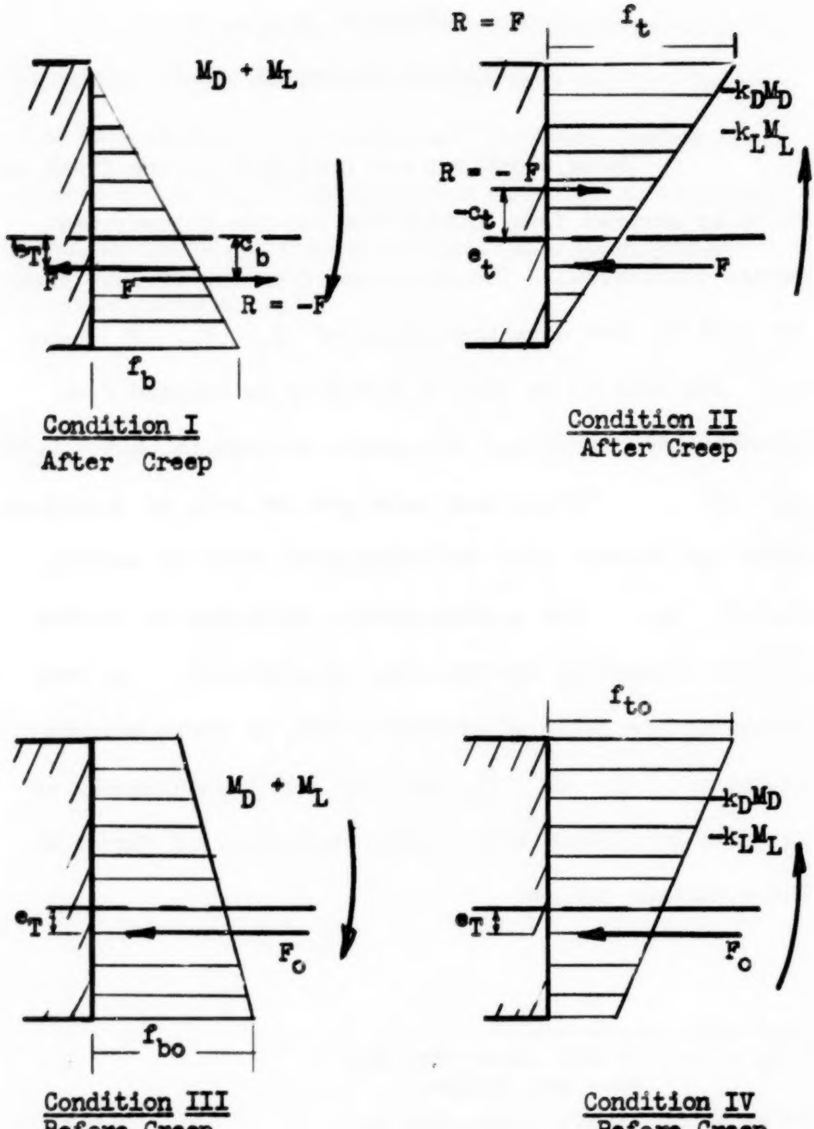

FIG. 12 - STRESS CONDITIONS For Live and Dead Load Moments of Varying Sign.

CASE I:

It may be shown that when:

$$\left.\begin{array}{l} k_D + \alpha k_L > 0 \\ \text{and } \alpha > -1 \end{array}\right\} \begin{array}{l} \text{then } f_{bo} > f_b \\ \text{and } f_{to} > f_t \end{array}$$

In addition, determine if:

$$(Z_r - \eta)\ (1 + \alpha) + (k_D + \alpha k_L)\ (\eta\ Z_r - 1) \gtrless 0$$

(a) < 0; then $f_{to} < f_{bo}$

(b) > 0; then $f_{bo} < f_{to}$

(a) $$(Z_r - \eta)\ (1 + \alpha) + (k_D + \alpha k_L)\ (\eta\ Z_r - 1) < 0$$

$$f_{bo} > f_{to}$$

Hence equate $f_{bo} = f_{cp}$ and

$$Z_b = \frac{M_D\ (\eta + k_D) + M_L(\eta + k_L)}{\eta\ f_{cp}} \quad \ldots\ldots\ (1)$$

$$Z_t = \frac{M_D\ (\eta + k_D) + M_L\ (\eta + k_L)}{\eta\ Z_r\ f_{cp}} \quad \ldots\ldots\ (2)$$

$$F_o = \frac{A_c\ [M_D\ (1 + k_D) + M_L\ (1 + k_L)]}{\eta\ (Z_b + Z_t)} \quad \ldots\ (3)$$

$$e_T = -\frac{M_D + M_L}{F_o} + c_b = \text{Lower limit} \quad \ldots\ldots \quad (4)$$

$$e_T = \frac{k_D M_D + k_L M_L}{F_o} - c_T = \text{Upper limit} \quad \ldots\ldots \quad (5)$$

and:

(b) $(Z_r - \eta)\ (1 + \alpha)$

$+ (k_D + \alpha k_L)\ (\eta Z_r - 1) > 0; \quad f_{to} > f_{bo}$

Hence $f_{to} = f_{cp}$

$$Z_t = \frac{M_D (1 + \eta k_D) + M_L (1 + \eta k_L)}{\eta f_{cp}} \quad \ldots\ldots \quad (6)$$

$$Z_b = Z_r \frac{[M_D (1 + \eta k_D) + M_L (1 + \eta k_L)]}{\eta f_{cp}} \quad \ldots \quad (7)$$

Again use equations (4) and (5) for limits of e_T.

Use equation (3) with equations (6) and (7) for determination of F_o.

CASE II:

If $k_D + \alpha k_L > 0$ and $\alpha < -1$, then $f_{bo} > f_{b\eta}$, then $f_{t\eta} > f_{to}$

The following equation is for determining the greater stress:

$$\eta(1 - Z_r)(\alpha + 1) + (1 - \eta Z_r)(k_D + \alpha k_L) \lesseqgtr 0$$

$$> 0; \quad f_{bo} > f_{t\eta}$$

$$< 0; \quad f_{t\eta} > f_{bo}$$

(a) $f_{bo} > f_{t\eta}$; $f_{bo} = f_{cp}$

Z_b from equation (1).

$$Z_t = \frac{M_D (1 + k_D) + M_L (1 + k_L)}{\eta f_{cp} Z_r} \quad \ldots\ldots\ldots \quad (8)$$

Use equations (4) and (5) for the limits of e_T.

Use equation (3) with equations (1) and (8) for evaluating F_o.

(b) $f_{t\eta} > f_{bo}$; $f_{t\eta} = f_c$

$$Z_t = \frac{M_D (1 + k_D) + M_L (1 + k_L)}{f_{cp}} \quad \ldots\ldots\ldots \quad (9)$$

$$Z_b = Z_r \frac{M_D (1 + k_D) + M_L (1 + k_L)}{f_{cp}} \quad \ldots\ldots \quad (10)$$

$$F_o = \frac{A_c f_{cp}}{\eta (1 + Z_r)} \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

<u>CASE III</u>:

$$\left.\begin{array}{l} \text{If } k_D + \alpha k_L < 0 \\ \text{and for } \alpha > -1 \end{array}\right\} \begin{array}{l} \text{then } f_{b\eta} > f_{bo} \\ \text{then } f_{to} > f_{t\eta} \end{array}$$

$$(Z_r - \eta)\ (1 + \alpha)$$
$$+\ \eta (Z_r - 1)\ (k_D + k_L) \lesseqgtr 0$$

(a) $> 0; \quad f_{to} > f_{b\eta}$

(b) $< 0; \quad f_{b\eta} > f_{to}$

(a) $f_{to} > f_{b\eta}$; $\quad f_{to} = f_{cp}$

Use all equations given in CASE I (b).

(b) $f_{b\eta} > f_{to}$; $\quad f_{b\eta} = f_{cp}$

Use equation (9) replacing Z_t by Z_b.

$$Z_t = \eta \frac{M_D (1 + k_D) + M_L (1 + k_L)}{Z_r\ f_{cp}} \quad \ldots\ldots \quad (12)$$

$$F_o = \frac{A_c \, Z_r \, f_{cp}}{\eta (Z_r + 1)} \quad \dots\dots\dots\dots\dots\dots \quad (13)$$

Use equations (4) and (5) for limits of e_T.

<u>CASE</u> <u>IV</u>:

$$\left.\begin{array}{l} \text{If } \; k_D + \alpha k_L < 0 \\ \quad \text{and for } \alpha < -1 \end{array}\right\} \begin{array}{l} \text{then } \; f_{b\eta} > f_{bo} \\ \text{then } \; f_{t\eta} > f_{to} \end{array}$$

$$\text{For } Z_r < 1; \quad f_{b\eta} > f_{t\eta}$$

$$Z_r > 1; \quad f_{t\eta} > f_{b\eta}$$

(a) $f_{t\eta} > f_{b\eta}$; $\quad f_{t\eta} = f_{cp}$

Use equation (9) for Z_t.

$$Z_b = Z_r \frac{M_D (1 + k_D) + M_L (1 + k_L)}{f_{cp}} \quad \dots\dots\dots \quad (14)$$

$$F_o = \frac{A_c \, f_c}{\eta (Z_r + 1)} \quad \dots\dots\dots\dots\dots\dots \quad (15)$$

Use equations (4) and (5) for limits of e_T.

(b) $f_{b\eta} > f_{t\eta}$; $f_{b\eta} = f_{cp}$

Use equation (9) replacing Z_t by Z_b.

$$Z_t = \frac{M_D (1 + k_D) + M_L (1 + k_L)}{Z_r \; f_{cp}} \quad \ldots\ldots\ldots\ldots \quad (16)$$

Use equation (13) for F_o.

Use equations (4) and (5) for limits of e_T.

APPENDIX B

The simultaneous equations:

$$e_T^{Ij} = e^{Ij} + \frac{(M_e^{Ij})_i}{F^{Ij}}$$

upon substitution of the values of the indeterminate moments $(M_e^{Ij})_i$ assume the following form:

$$e_T^{Ij} = a_{Ij}^{I1} \; e^{I1} + a_{Ij}^{I2} \; e^{I2} + \ldots . \; a_{Ij}^{Ij} \; e^{Ij} + \ldots$$

$$+ a_{Ij}^{In} \; e^{In} + a_{Ij}^{II1} \; e^{II1} + a_{Ij}^{II2} \; e^{II2}$$

$$+ \ldots a_{Ij}^{IIj} \; e^{IIj} + \ldots + a_{Ij}^{IIn} \; e^{IIn}$$

$$+ \ldots$$

It should be noted that the coefficients a_{Ij}^{IIp} denote the influence of the determinate prestressing moment $F^{IIp} \; e^{IIp}$ at section p of member II, as denoted by the superscript, on the thrust eccentricity at section j of member I, denoted by the subscript. In most cases the coefficient a_{Ij}^{Ij} is much larger than the other coefficients and a first approximation for the cable ordinates may be obtained from the following expressions:

$$(e^{Ij})' = \frac{e_T^{Ij}}{a_{Ij}^{Ij}}$$

$$(e^{IIj})' = \frac{e_T^{IIj}}{a_{IIj}^{IIj}}$$

The above values are substituted in the equations for the thrust eccentricities and values ... $(e_T^{Ij})'$ .... $(e_T^{IIj})'$ .... are obtained.

In most cases the values e_T^{Ij} are chosen arbitrarily consistent with the upper and lower limits of the thrust eccentricities as determined from the equations of Appendix A. In several design examples on continuous prestressed concrete structures, it was found that the majority of the values $(e_T^{Ij})'$ ... $(e_T^{IIj})'$ .... were between the required limits. Depending upon the case, several of the first approximation values $(e^{Ij})'$ are adjusted and a new set of the thrust eccentricities $(e_T^{Ij})''$ .... $(e_T^{IIj})''$ .... is obtained. The procedure is repeated until all the thrust eccentricities are within the design requirements. Usually the second or third approximations render satisfactory re-

sults. The solution may be set up in a tabular form and the approximations are made by entering the algebraic changes to the thrust eccentricities due to the successive adjustments of the cable ordinates. A set of fifteen equations may readily be solved in a matter of two hours by this method.

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